Suitability and Optimality in the Asset Allocation Process

by

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Abstract

Suitability is a legal concept that refers to the propriety of the match between the individual and his or her portfolio. Financial advisors and investment companies employ numerous models to profile investors and then recommend a suitable asset allocation. However, there is no guarantee that the recommended asset allocation is also optimal in a mean-variance sense. We develop a model of suitability using the Analytic Hierarchy Process (AHP) to create unique asset allocations for individual investors based on their personal attributes. We then compare the mean-variance performance of these suitable portfolios with independent portfolios generated using traditional mean-variance optimization (MVO) methodology. Our results indicate that the AHP and MVO approaches yield portfolios with risk-return attributes that are not significantly different. The AHP portfolios are more likely to underperform the MVO portfolios for individuals with very high risk tolerance. We find that minor alterations to the AHP model can further minimize any distinction from a pure MVO portfolio. Finally, we argue that sequential application of the two approaches provides superior results when compared to those generated by AHP alone.

Suitability and Optimality in the Asset Allocation Process

Introduction

Financial advisors, large and small, are frequently asked to recommend an asset allocation for individual investors. There are numerous examples of hard copy and web based programs assisting individual investors to select a reasonable asset allocation and to offer mutual funds that will allow execution of the proposed strategy. Most of these models use a questionnaire with the primary purpose of ascertaining the investor's level of risk tolerance. Such questionnaires may also probe an investor's time horizon and strategic elements that distinguish this investment from others already in place.

Responses to these questions are scored and scores are accumulated. Typically, this approach results in a recommendation of one fund from a vector of funds that differ in terms of risk. Others may recommend several asset-class specific funds collectively providing the desired asset allocation.

These models suffer from several problems. First, while primarily focusing on risk tolerance, they miss other relevant attributes to the construction of a suitable portfolio. Second, by using a uni-dimensional scale, they create a "flat" set of choices that are linear in risk. Third, they drive the result toward a relatively small set of choices. Fourth, the recommended asset allocation may be suitable, but not optimal. Many combinations of funds may produce a suitable investment, but only one will be optimal.

In this paper, we develop two distinct and useful contributions:

- 1. We propose and construct a robust methodology to develop a suitable asset allocation for individuals that matches an investor's goals, risk tolerance and financial situation. These portfolios are composed of mutual funds that represent a wide range of distinct asset classes. Suitable portfolios constructed from assets that are ranked for suitability on a number of different investor profile dimensions. Suitable portfolio weights are in proportion to the asset suitability, as determined by subjective suitability rankings and the Analytic Hierarchy Process, or AHP, algorithm.¹
- 2. We evaluate our suitable recommendation for mean-variance optimality. Efficient portfolios are portfolios of minimum risk for a given level of expected return.² Modern portfolio theory, however, suffers from the problems of estimation error that often causes investment professionals to consider optimal portfolios to be unsuitable for their purpose.³ Although this study uses techniques designed to reduce the impact of estimation error, the paper's main goal is to reconcile suitable and optimum portfolios by adjusting AHP and mean variance parameters. This is done by minimizing the expected return penalty of suitable portfolios and by varying the risk tolerance to reduce the discrepancies between the two techniques. The key to this iterative process is to maintain or improve the plausibility of judgmental factors by incorporating knowledge gained by running the complementary process.

Distinguishing Suitability from Optimality

Suitability is the quintessential problem that faces the financial advisor. Once a client's personal and financial situation is evaluated, a policy asset allocation between equities, fixed income and cash equivalents can be determined. This allocation can be further articulated by subdividing these asset categories, including an international dimension, and considering real assets (real estate, commodities, etc.) or other instruments (derivatives). A suitable portfolio is one in which the assets held are appropriate to the investment objectives, financial needs, and level of sophistication of the client.

However, a suitable portfolio developed for an individual need not be optimal in a meanvariance sense. Even if we fulfill a prescribed asset allocation with the best category specific funds (or combinations of funds), there is no guarantee that the resulting portfolio will be the highest return available at the given level of expected risk. Likewise, a mean-variance efficient portfolio, with a reasonable level of risk may not be suitable for a particular investor.

Modeling Suitability with the AHP⁴

The AHP has three basic steps. It begins by decomposing the overall goal (Suitability) into a number of factors and subfactors. The goal itself represents the top level of the hierarchy. Major factors comprise level two, subfactors make up level three, and so on. Some of these factors (and subfactors) overlap and others may lead to inconsistent recommendations when viewed in isolation. Many factors must be evaluated subjectively. The Analytic Hierarchy Process is a methodology well suited to such an evaluation. We have modeled suitability using a series of factors drawn from NYSE Rule 405, the "know your customer" rule. The bottom level of the hierarchy consists of the decision alternatives. In this case, asset classes represent these alternatives.

Once modeled, the second basic step of the AHP begins. Within each level of the hierarchy, the relative importance between each pair of factors (or among pairs of subfactors relating to a single factor) to the overall goal is evaluated. A nine-point scale is commonly used for these evaluations. For example, if comparing factor A to factor B, a score of 1 indicates that they are equally relevant to the evaluation of suitability and a score of 9 indicates that B is of little significance relative to A. All scores can be assembled in a pairwise comparison ma trix with 1s on the diagonal (e.g., A to A is 1) and reciprocal scores in the lower left triangle (e.g., if A to B is 5, then B to A is 1/5). Pairwise comparisons generated for the upper levels of the hierarchy contain expert opinion regarding the relative importance of factors. This portion of the hierarchy is not influenced by varying attributes of individual investors and therefore can be "hard wired" into the system. However, pairwise comparisons must be reevaluated at the bottom level of the hierarchy to assess relative suitability of decision choices (asset classes) and each subfactor (investor attributes) in the level just above. These comparisons capture the differences between individual investors.

The third and final step in the AHP requires evaluation of the pairwise comparison matricies using measurement theory. A standardized eigenvector is extracted from each

matrix, allowing us to assign weights to factors, subfactors, and ultimately asset classes. These weights allow us to assemble a suitable asset allocation for an individual investor.

The Suitability Hierarchy

The specific hierarchy developed here consists of four levels. The upper levels of the hierarchy are solved to produce a weighting scheme that will determine the relative importance of each factor or subfactor in determining the suitable portfolio. At the lowest level of the hierarchy, asset classes are evaluated to produce a portfolio that is suitable for one dimension (a single investor attribute) of the problem. The final suitable portfolio is chosen by combining the local weights derived for each asset class and the weights produced by the higher levels of the hierarchy.

As indicated above, our model of suitability borrows heavily from the NYSE's Rule 405. The hierarchy is illustrated below in Figure 1:



Figure 1 Analytical Hierarchy

	IS	IO	IE	Weight
IS	1	1	3	0.4286
Ю	1/3	1	3	0.4286
IE	1/3	1/3	1	0.1429

Table 1 Pairwise Comparisons for Suitability Factors

The three Level 2 factors are further divided into a total of 17 subfactors (7, 7, and 3 respectively). This requires the development of three additional pairwise comparison matricies. The first, for subfactors of Income and Savings, is given below. Again, the rightmost column contains the normalized eigenvector which is used to consolidate all pairwise comparisons into a single weight vector.

	INCOME	SOURCE	SAVINGS	SVG. RATE	CASH HOLD	FI HOLD	EQUITY HOLD	WEIGHTS
INCOME	1	5	1/2	3	6	4	2	0.2399
SOURCE	1/5	1	1/6	1/3	2	1/2	1/4	0.0448
SAVINGS	2	6	1	4	7	5	3	0.3543
SVG. RATE	1/3	3	1/4	1	4	2	1/2	0.1036
CASH HOLD	1/6	1/2	1/7	1/4	1	1/3	1/5	0.0312
FI HOLD	1/4	2	1/5	1/2	3	1	1/3	0.0676
EQUITY HOLD	1/2	4	1/3	2	5	3	1	0.1587

Table 2 Pairwise comparisons for Income & Savings Subfactors

We repeat this process for the remaining two subfactor pairwise matricies. This completes construction of the "hardware" portion of the model. The relative weights of factors and subfactors result from expert evaluation of the importance of each pair of elements with respect to suitability. While other experts may disagree with our comparison analysis, the resulting eigenvectors should remain fixed for all individual investors. This allows the AHP to incorporate a combination of measurable and subjective elements of a complex decision in a consistent manner over repeated trials.

It is also worth noting that an n-factor pairwise comparison matrix requires n(n-1)/2unique pairwise comparisons. Thus far we have evaluated matricies of rank 3, 7, 7, and 3. This represents a total of 48 individual pairwise comparisons. While this is easily accomplished, a more daunting task lay ahead. Now, each of the 17 subfactors represents a unit of information collected from an investor. Every subfactor must be individually connected to a pairwise comparison matrix of the 18 asset classes. This means that an individual response to a subfactor such as Age, will require 153 unique comparisons. Further, if "Age" is a categorical variable with 5 possible responses, we now must generate distinct matrices for each, a total of 765 comparisons for a single subfactor. When the entire array of 17 subfactors is similarly evaluated, we could require as many as 8,415 pairwise comparisons to cover any combination of categorical responses to questions! Clearly, there is a need for a shortcut!

We reduce the demands on the decision maker by focusing on a single row of each pairwise comparison matrix and then using a simple algorithm to generate consistent comparisons for subsequent rows. This requires the alternatives to be rank ordered and mapped onto the same 9-point scale described above. Once completed, lower rows can be derived maintaining similar relative rankings for remaining comparisons. This is a compromise from the full information form of the AHP, but makes construction of the model far less demanding.

Additional Data Needs

Asset Class Proxies

The ideal approach to assembling our asset class proxies requires identification of specific indicies for each our 18 asset classes. However, the index approach causes two problems. First, several of our asset classes are very narrow and do not match well with a precise and available index. Second, many of the asset class indicies we identified are not investable. To avoid these problems, we identify one mutual fund to proxy for each fund asset class as defined by the Micropal database of approximately 7000 mutual funds and indices. We identified the proxy for the asset class using the following criteria:

- 1. The fund requires ten years of return statistics in order to estimate its historic performance, risk, and contribution to risk due to correlation with other asset classes.
- 2. The fund is the 75th percentile of its asset class based on the ten year Sharpe ratio. If the 75th percentile fund satisfies the two criteria below, it is chosen as the proxy for the asset class. This ranking represents the ability of investment advisors to choose "good" mutual funds and judges funds on both return and risk metrics⁵.
- 3. Its CAPM return expectation is consistent with the consensus return characteristics of the asset class. In other words, the order shown in Table 1 was accepted as a prior expectation and funds that did not preserve this order were not chosen as suitable. In other words, the proxy Aggressive Growth Equity fund has a higher return than the proxy Growth fund. Similarly, the proxy High Yield Bond fund has a higher return than the proxy High Quality Bond fund. Only of a few funds failed this test. These funds were replaced by the next closest fund to the 75th percentile.
- 4. The asset class style should be consistent with the asset classification.⁶ Style analysis determines what allocation of component asset classes best matches the monthly performance of a portfolio. If a fund's predominant weighting does not match its classification or the fund has a high tracking error, the funds were replaced by the next closest fund to the 75th percentile. Style analysis identified only one fund as inappropriate to represent the asset class.

Predominant Asset Class	Fund Type	Default Fund	Return	Std. Dev.	Sharpe*	Beta
Aggressive Equity	Precious Metals	Handy&Harman, Gold	-4.40	11.40	-0.87	-0.06
Cash	Money Market, Gov	Dreyfus 100% US Treas	4.70	0.32	-2.50	0.00
Cash	Money Market, taxable	ONE Fund Money Mkt	4.60	0.31	-2.90	0.00

Table 3 Funds Chosen for Each Asset Class

Cash	Money Market, tax free	Prudential Tax Free	3.90	0.16	-10.00	0.00
Aggressive Bond	Mortgage Backed	Fidelity Mortgage Sec	8.60	3.22	0.96	0.17
Conservative Bond	Government Bonds	Strong Government Sec	9.31	3.88	0.98	0.18
Conservative Bond	Bond-High Quality	Fidelity U.S. Bond Index	8.86	4.11	0.82	0.23
Aggressive Bond	Bond-High Yield	Fidelity Advisor High Yield	12.66	6.86	1.04	0.31
Aggressive Bond	Global Bonds	Paine Webber Global Income	7.92	4.92	0.49	0.32
Aggressive Bond	Convertible Bonds	Value Line Convertible	11.92	9.40	0.68	0.56
Conservative Equity	Utility	Prudential Utility/A	13.86	9.19	0.91	0.67
Conservative Equity	Income	Riverfront Income Equity	15.95	11.94	0.88	0.88
Conservative Equity	International Equity	Templeton Foreign/1	14.05	9.65	0.89	0.94
Conservative Equity	Growth and Income	Vanguard Growth and Income	17.48	13.40	0.89	1.01
Aggressive Equity	Growth	MFS Large Cap Growth	18.71	15.23	0.87	1.07
Aggressive Equity	Small Cap	Janus: Smal Cap	22.36	17.40	0.97	1.19
Aggressive Equity	Aggressive Growth	Spectra Fund	20.74	14.71	1.04	1.31
Aggressive Equity	Specialty	T Rowe Price Sci&Tech	24.33	19.01	0.99	1.37

Data from the Investor via Questionnaire

The lowest level of the hierarchy is the most labor intensive to implement. This is the point where individual differences among investors must be articulated. Pairwise comparisons leading to suitable asset allocation must be generated for each question (subfactor) and response level. We elicit responses to 17 questions, most requiring selection of one of five response categories, with the level '5' representing the most risk tolerant response and the level '1' representing the most risk averse response. A copy of the questionnaire is included as an Appendix. Each of the 76 possible responses in the questionnaire is mapped to a "suitability vector" containing relative weightings of the asset classes from 1 to 9, with 1 being the most suitable class and 9 being the least suitable asset class for each response for each question. This requires more than 1,368 asset class suitability assessments for 18 asset classes and 17 questions with a total of 76 distinct responses.

To determine if our weightings and classification were producing reasonable results, we created five hypothetical investors that essentially spanned the types of investors that would use this service.

Investor 5 is a typical aggressive or high net worth investor. For these investors, '5' is the average response for each question and aggressive equities are most suitable assets. Money market and bond investments are less suitable assets for this type of investor. Asset classes with a beta near 1.0 are considered maximally suitable.

Investor 4 is a moderately aggressive investor with income and a net worth that is not quite in the most aggressive quintile. For this type of investors, '4' is the average response for each question and aggressive bonds and conservative equities are the most suitable asset classes. Initial suitability estimates were interpolated between Investor 5 and Investor 3, described below.

Investor 3 is a typical, middle income and net worth investor. Income, savings, and age represent the middle quintile of investors today. For these investors, '3' is the dominant response for each question. Conservative and aggressive bonds and conservative equities are the most suitable asset classes. For this case, the initial suitability is first estimated by interpolating between the extreme cases (Investor 5 and Investor 1). This created an assignment of moderate suitability to all asset classes. This assignment was then modified by:

- Increasing the suitability of moderate beta (such as bonds and utility stocks) assets
- Decreasing the suitability of the extreme beta stocks.

Investor 2 is a conservative investor with moderate net worth and income. For these investors, '2' is the average response for each question. For these investors, the money market and conservative bonds are the most suitable asset classes. Initial suitability estimates were interpolated between Investor 3 and Investor 1, described below. Government bonds were considered most suitable for this conservative investor.

Investor 1 represents the extreme risk-averse investor. For these investors, '1' is the dominant response for each question. These investors need to hang on to their assets and have very little flexibility to handle any decline in their savings, even if it means giving up the chance at a higher return from their investors. For these investors, only the money market and the most conservative bonds are suitable asset classes. All other asset classes were considered to have the highest level of unsuitability. This suitability characteristic was assigned to all the suitability factors at the lowest level of the suitability hierarchy.

Initial Results from the AHP

After generating 76 suitability vectors covering the individual questionnaire responses and the eigenvector weights associated with them, we assemble all weights using our five hypothetical investors to evaluate the overall suitability of our asset allocation.

For example, if we evaluate Investor 4's response regarding the level of her income, we get the following suitability vector.

Table 4: Example of Pairwise comparisons and Resulting Weights of Asset Classes Resulting from a Single Questionnaire Response

Precious Metals	Money Mrket, Gov	Money Mrket, taxable	Money Mrket, tax free	Gov'nt Bonds	Mort - gage Backed	Bond- High Quality	Bond- High Yield	Global Bonds	Convrt Bonds	Utility	Income	Int'n'l Equity	Growth &Incom e	Growth	Small Cap	Aggres Growth	Special
8	8	8	8	1	3	2	5	5	5	4	3	2	1	1	2	3	4
0.0100	0.0100	0.0100	0.0100	0.1287	0.0559	0.0854	0.0253	0.0253	0.0253	0.0372	0.0559	0.0854	0.1287	0.1287	0.0854	0.0559	0.0372
0.0036	0.0036	0.0036	0.0036	0.0456	0.0198	0.0303	0.0089	0.0089	0.0089	0.0132	0.0198	0.0303	0.0456	0.0456	0.0303	0.0198	0.0132
0.0015	0.0015	0.0015	0.0015	0.0195	0.0085	0.0130	0.0038	0.0038	0.0038	0.0056	0.0085	0.0130	0.0195	0.0195	0.0130	0.0085	0.0056

The integer numbers in the top row indicate the relative suitability of each asset class for an individual with savings in excess of \$500,000. Clearly, we'd like to know more about this person, but based on this single data item, we find Government Bonds, Growth and Income Equity, and Growth Equity classes to be most suitable. Precious Metals and Money Market classes are the least suitable. The three rows following the pairwise comparisons contain:

- 1. The normalized eigenvector weights from the full pairwise comparison matrix derived from the asset class rankings. This represents the local weight of the asset classes.
- 2. Weights from (1) multiplied by the weight of the Savings subfactor within the Income and Savings factor (35.43%).

3. Weights from (2) multiplied by the weight of the Income and Savings factor within the overall goal of Suitability (42.86%). The upper level weights are illustrated for all factors and one subfactor in Tables 1 and 2.

For example, Growth Equities are assigned a 12.87% local weight. If we only knew this investor's savings level, we'd assign this proportion to this asset class. However, when we account for the 16 other responses, the local weight must be conditioned by the higher-level weights. Hence, this single response contributes 1.95% to our final allocation in Growth Equities. While this number appears small on its own, repeating this process for all other responses (subfactors) will generate their own global contributions to each asset class allocation. When accumulated, we have assembled a global allocation that reflects our breakdown of the suitability process and the salient attributes of the individual investor.

For simplicity in the ensuing discussion, we consolidate our 18 asset classes into 5 broader categories: Money Market, Conservative Fixed Income, Aggressive Fixed Income, Conservative Equity, and Aggressive Equity. The ultimate asset allocation recommendation for our 5 hypothetical investors is summarized in the following figure.



Figure 2 Asset Allocation Using AHP for Five Hypothetical Investors

As one would expect, Investor 5's portfolio is predominantly equity, reflecting his high degree of risk tolerance. On the opposite extreme, Investor 1 has nearly half of his assets in money market instruments and most of his remaining funds in fixed income. The other investors fall rather predictably in between. Ex-post, these asset allocations appear to match up reasonably well with their owners.

However, there has been no explicit attempt to create an allocation that is also meanvariance optimal. The next section begins with a brief review of the mechanics of meanvariance optimization (MVO). We then proceed to evaluate the AHP/Suitable asset allocations in an MVO context.

Mechanics of Mean Variance Optimization

While the asset allocations provided above may suitable from a legal perspective, it would be easier to justify such allocations if they are not significantly different from portfolios that are optimal. Modern Portfolio Theory blends expectations for the returns and risks from different combinations of investments to find a diversified portfolio that offers the most attractive relationship between return and risk. Modern Portfolio Theory defines efficient portfolios assuming the expected returns and contribution to risk of each portfolio asset. There are many ways to compute this expected return and the risk factors, and the estimation technique has profound implications on the efficient frontier. Figure 3 shows the impact of estimating the returns and covariances using the following three techniques:

- 1. Estimate returns and covariances historically.
- 2. Estimate returns historically and use Bayes-Stein adjustment to statistics to estimate returns and risk factors.⁷ Bayesian statistics that assumes a strong prior assumption about the process's distribution and then modifies these beliefs upon adding data.
- 3. Estimate historic β and assume that historic β and the Capital Asset Pricing Model can estimate expected returns.



Figure 3 Comparison of Efficient Frontier Using Different Return and Risk Matrix Estimation Techniques

For this study, we use the CAPM model to estimate expected returns. Capital market theory indicates that the expected excess return of a financial asset is proportional to the product of excess return of "the market" and the β of the asset's returns to the market

returns.⁸ Excess returns are defined as the return above the risk free rate, such as the return on a 3-month U.S. treasury bill. Therefore, the expected return of an asset class is:

$$r_i = r_f + \beta_i (r_m - r_f)$$

where:

$$\begin{split} r_i &= \text{Expected return of asset class i.} \\ r_f &= \text{risk free rate} \\ \beta_i &= \text{risk as measured by beta for asset class i; } \beta_i &= \rho_{i,m} \sigma_i / \sigma_m \\ \rho_{i,m} &= \text{correlation between the market returns, and asset class i's returns.} \\ \sigma_i &= \text{volatility of asset class i's returns.} \\ \sigma_m &= \text{volatility of the market return.} \\ r_m &= \text{return of "the market"} \end{split}$$

The U.S. large capital equity total return index (i.e. the S&P 500) is often considered a suitable proxy for "the market" when estimating expectations for individual stocks. For asset allocation, however, this is a very incomplete market proxy for the purposes of this application. The market should be defined to include all domestic and foreign financial assets -- stocks, bonds, real estate, other tangibles.

To correctly use the CAPM, one must specify the world market and use a long enough time series to accurately estimate β for each asset class. We chose the following weighted index to approximate the world market for financial assets:

- 30% U.S. Equities (70/15/15 large, mid and small capitalization)
- 20% Domestic Bonds
- 30% Non U.S. Stocks, e.g., Morgan Stanley EAFE
- 20% World Bonds

This market has an excess return over the U.S. risk free rate of about 6%, and has an annualized risk (standard deviation) of about 8% over the last 10 years. Analyzing this market for a twenty-year period indicates that it is very close to the point of maximum Sharpe ratio and has similar risk-return characteristics. In other words, this definition of the market appears to confirm capital market preconceptions while being reasonably stationary.



Figure 4: World Market **b** vs. Return

The upper scatter plot and trend line in Figure 4 shows a very strong relation between β and return for the last ten years. The lower line is the CAPM line assuming that historic β represents asset risk class and the slope of the CAPM line is 6%. This line has an intercept (risk free rate) of 5%, which is close to both the t-bill average over the last 10 years and the risk free rate for 1998. Figure 4 shows that using only the last 10 years to estimate market returns (the value of the dotted trend line at $\beta = 1.0$) seriously overestimates the return of the market in comparison to the CAPM expectation. Using the conservative CAPM expectations will balance the optimistic decision to employ the 75th percentile fund rather than the median fund as the asset class proxy.

It is also possible to predict asset class returns using more fundamental prediction techniques. These techniques, however, will be unstable over time despite being more sophisticated. Other examples of tactical asset allocation techniques include using the current yield to estimate the expected return for each fixed income asset class or using fundamental analysis to estimated the return of equity classes. These practices are essentially "market timing", a difficult practice and not suitable for the majority of individual investors.

Determining an Individual's Risk Tolerance

The investor's choice of a portfolio is limited by the efficient frontier, but a rational investor will choose to be at a specific point on the efficient frontier. We define the risk acceptance parameter, or RAP as

$$\Delta \sigma_{\rm p}^{2} / \Delta E(R_{\rm p})$$

The RAP is used to trade off expected return against risk. It is the inverse of the slope of the efficient frontier. A higher RAP indicates greater risk tolerance and preference for a more aggressive portfolio. A simple way to develop an MVO portfolio for our 5 hypothetical investors is to create a profile for each based on their questionnaire responses employed in the AHP process described previously. In all cases, the optimum portfolios and portfolio performance are estimated assuming CAPM return expectations and a risk matrix estimated using Bayesian adjustment. CAPM returns and asset covariances are measured using ten years of monthly returns for the fund chosen to represent each asset class. The RAP estimates for the 5 investors are described below.

- 1. Investor 5 has an average response near 5, and is assigned a RAP of about 50. Higher RAPs than 50 will result in portfolios with extremely highly concentrations of high beta assets. This type of investor would be better off extrapolating the leveraged extension of the capital market line, by combining a well balanced portfolio with equity futures and options that represent a leveraged position in the underlying index.
- 2. Investor 4 has a questionnaire average response near 4 and is assigned an initial RAP of about 40, with a range between 30 and 45.
- 3. Investor 3 has an average response near 3 and is assigned an initial RAP of about 25, with a range between 15 and 35.
- 4. Investor 2 has an average response near 2 and is assigned an initial RAP of about 15, with a range between 9 and 20.
- 5. Investor 1 has an average response near 1 and is assigned an initial RAP of about 7, with a range from 3 to 10.

Once a RAP has been determined, the optimal asset allocation can be derived. Figure 5 shows the MVO results for the broad asset class aggregates. Just as the AHP derived weights illustrated in Figure 2, are "covariance blind", the MVO results are "suitability blind."



Figure 5: Asset Allocation Using MVO for Five Hypothetical Investors

A comparison of Figures 2 and 5 provide for an interesting comparison between the two approaches. Casual observation of the two figures suggests that the estimated allocations are quite similar. This should be a source of comfort for both financial advisors and "quant" portfolio managers! To a reasonable degree, MVO implicitly addresses suitability and AHP implicitly considers optimality.

Reconciling Suitability and Optimality

We next use an iterative technique to evaluate and ensure that AHP and MVO portfolios have consistent asset allocation and mean variance performance by adjusting the AHP and MVO parameters to reduce the following unsuitability penalties:

- 1. How far below the efficient frontier is a suitable portfolio?
- 2. How far is the risk of the MVO portfolios from the risk of the AHP portfolios?

Figure 6 shows that the AHP portfolios is almost certainly inside the efficient frontier's uncertainty band that results from the expected returns vector and covariance matrix estimation errors.



Figure 6: Comparison of AHP and Mean Variance Portfolios Risk Return Performance

AHP portfolios are subjective judgments of suitability. Comparing AHP and optimum portfolios allow us to (1) assess the absolute disparity of the two approaches and (2) refine suitability judgments to reduce that disparity. Because the judgments made in determining suitability reflect considerable knowledge embedded in the AHP, it is justifiable to adjust the RAP to ensure that MVO portfolio's risk level converges to the risk level of the suitable portfolios. During each iteration, the weights were adjusted in a fashion that preserved the character of the suitability matrices in both dimensions:

- 1. Suitability weights are adjusted such that asset classes with similar expected returns will have similar suitability for every factor (question) considered.
- 2. Suitability weights are adjusted such that the change in suitability for each level of risk tolerance preserves the basic characteristics of the initial curves. Types of changes would be:
 - 2.1. The falloff in suitability from the most suitable asset class can increase or decrease.
 - 2.2. The beta of the most suitable asset class can increase or decrease.
 - 2.3. The suitability of asset classes at the extremes (money market or aggressive equities) could change.

The most detailed way to compare and reconcile suitable and optimal portfolios is to examine the implied returns of suitable portfolios with the returns estimated by the CAPM model. Implied returns are derived under the assumption that the AHP weights

are optimal. In other words, rather than using expected asset class returns and covariances and solving for the optimal weights, we assume the weights are optimal and extract the expected asset returns that would make this true. If the implied returns are close to the expected returns, then we can also conclude that the AHP weights are close to optimal. Figure 6 shows that the implied returns and estimated returns track reasonably closely for each asset class and hypothetical investor. Analysis of results illustrated in Figure 6 indicates that, while there are differences between implied and expected returns, they are not statistically significant. This holds for every asset and investor combination, allowing us to conclude that the implied return from the AHP weights are reasonable proxies for expected asset class returns.





A New Role for MVO

In the previous section, we describe a technique to "nudge" the AHP-derived suitable portfolio toward optimality. Clearly, we can only adjust the AHP model a certain amount before we compromise the goal of suitability. At some point, we must make a recommendation to our client and that recommendation should be our "best" suitable asset allocation. Our analysis suggests that, at least for our hypothetical investors, the recommended asset allocation has risk and expected return characteristics that are not significantly different from those generated by the MVO approach.

If the AHP model now produces results that are essentially optimal, do we need an optimizer in our asset allocation process? The answer is yes. Consider two possible outcomes associated with the development of suitable asset allocations for non-hypothetical clients. First, it is possible that some investors will require a suitable asset allocation that is materially different from an optimum portfolio with similar risk.

Second, an investor's current portfolio may be very different from our recommended suitable asset allocation.

In both cases, we use the AHP-derived asset allocation as a benchmark in terms of risk and return. Next, we run an optimization that penalizes tracking error versus the suitability benchmark. In the first case, an optimizer could identify alternate asset allocations that have superior risk/return profiles and fall within a certain level of tracking error. In the second case, the investor's current portfolio already has tracking error versus the suitable portfolio. An optimization process allows us to identify alternative portfolios that behave more like the suitable portfolio (i.e., less tracking error). In either case, the optimization process can be constrained to limit trading costs or to set bounds for asset class exposures. MVO's role is to improve investment performance while retaining or improving the suitability of the recommended asset allocation.

Summary

In this paper we have described a technique for building asset allocations that are both suitable and optimal. This is accomplished using the Analytical Hierarchy Process to derive an allocation that would be suitable for an individual investor. Using investor responses to a questionnaire, we can also derive the risk-return tradeoff, or RAP, for an individual. Combining this RAP with expected returns and a covariance matrix, we can determine the mean-variance optimal portfolio for this same individual.

We then extract the vector of asset returns that would render the suitable portfolio to be mean-variance efficient. By comparing these implied returns with the true expected returns on asset classes, we can assess our progress toward the dual goals of suitability and mean-variance efficiency. This process is repeated for a sample of hypothetical investors and then an iterative process is employed whereby modest alterations are made to the AHP comparison matrices. The objective is to nudge the weights in the suitable portfolio toward those in the optimal portfolio without compromising the integrity of the suitability process.

Our analysis indicates that minor changes in the AHP rule base can close the gap between the asset allocation considered suitable and the one that is solely optimal in a meanvariance sense. While additional work must be done to assess the effectiveness and flexibility of the suitability model for a wider sample of real investors, our results indicate that suitability and optimality can indeed coexist. Furthermore, the sequential application of these two approaches potentially provides superior asset allocation recommendations than either process generates individually.

End Notes

¹ The asset allocation issue has been previously examined using AHP. See Bolster, P., Janjigian, V. and Trahan, E., "Determining Investor Suitability Using the Analystical Hierarchy Process," *Financial Analyst Journal*, July/August, 1995 and Khasiri, S., Kamath, R., and Grieves, R., "A New Approach to Determining Optimum Portfolio Mix," *Journal of Portfolio Management*, Spring 1989.

- ² Markowitz, H., Portfolio Selection, *Cowles Monograph* 16, Yale University Press, 1959
- ³ An extensive analysis of the influence of estimation error on portfolio optimization can be found in Michaud, R., *Efficient Asset Management*, Harvard University Press, 1998
- ⁴ The AHP methodology is well explained in Saaty, J. K., "The Analytic Hierarchy Process," McGraw-Hill, 1980.
- ⁵ We considered both the median and top performing funds as proxy candidates. However, the 75th percentile fund has exhibited superior performance over a ten year period when compared to the median fund. The top fund in each category is prone to style inconsistency.
- ⁶ See Sharpe, W., "Asset Allocation: Management Style and Performance Measurement," *Journal of Portfolio Management*, Winter 1992 for a discussion of style analysis. Also see, DiBartolomeo, D., and Witkowski, E., "Mutual Fund Misclassification: Evidence Based on Style Analysis," *Financial Analysts Journal*, September-October 1997.
- ⁷ See Jorion, P, "Bayes-Stein Estimation for Portfolio Analysis," *Journal of Financial and Quantitative Analysis*, 1986.
- ⁸ Sharpe W., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* Vol 19, #3, September 1964.

Appendix: Investor Questionnaire

Personal and Income Information:

- 1. Age:
 - ____ 70 and Over
 - ____ 55 to 70
 - ____ 50 to 55
 - ____ 30 to 40
 - ____ Under 30
- 2. Number of Dependants:
 - ____ Quite a Few
 - ____ Five or Under
 - _____ Just the Two of Us
 - ____ Just Me

- 3. Household Income:
 - ____ Under 30
 - ____ 30 to 60
 - ____ 60 to 100
 - ____ 100 to 200
 - ___ Doing Fine
- 4. Income Source:
 - ____ Investment
 - ____ Most Investment
 - ____ Earned Investment
 - ____ Most Earned
 - ____ Earned
- 5. Tax Rate:
 - ____ I pay my fair share, not near the top
 - ____ At or neat the top, tax free funds make sense for me

Savings Information:

- 1. Savings Totals:
 - ____ Not Much
 - ____ \$50,000 to \$150,000
 - ____ \$150,000 to \$500,000
 - ____ More than \$500,000
 - ____ Doing Fine
- 2. Savings Rates:
 - ____ Not Much
 - ____ 5% to 10%
 - ____ 10% to 20%
 - ____ 20% to 40%
 - ____ Doing Fine

- 3. Cash and Money Market:
 - ____ More than 75%
 - ____ 50% to 75%
 - ____ 20% to 50%
 - ____ 5% to 20%
 - ____ Not Much
- 4. Fixed Income:
 - ____ Not Much
 - ____ 5% to 20%
 - ____ 20% to 50%
 - ____ 50% to 75%
 - ____ More than 75%
- 5. Stocks and Equity Mutual Funds:
 - ____ Not Much
 - ____ 5% to 20%
 - ____ 20% to 50%
 - ____ 50% to 75%
 - ____ More than 75%

Objectives

1. Time Horizon:

How long do you hope to hold your portfolio before you intend to start using up the principal? Reasons might include buying a first or more expensive house, paying for a child's college tuition or your retirement. Although we also use other factors to estimate how aggressive you want to be, the longer time you have before you grow your savings, the more you flexible you can be.

- ____ Five Years or Less
- ____ Ten Years
- ____ Fifteen Years
- ____ Twenty Years
- ____ Twenty Five Years or More

2. Investment Income Consumed:

What portion of your investment income do you use for household expenses? Remember, the higher this number, the less your wealth will grow over time and the higher the yield your assets will needs, which may force you to hold a portfolio with less expected future return.

____ Almost All

____ Most

____ About Half

____ Less Than Half

____ Not Much

Investing Experience

1. Money Market:

- ____ I have a small money market account or bank account.
- _____ More than 20% of my assets are in the money market or savings account.
- ____ I am very comfortable in the money market, which is why most of my assets are in the money market or savings account.
- 2. Fixed Income Bonds and Annuities:
 - _____ No fixed income or I own a small bond fund, but that is it.
 - I am somewhat comfortable with bond investments, it would be OK if more than 20% of my assets are in bonds or bond funds.
 - ____ I am very comfortable investing in bonds and fixed income funds.
- 3. Stocks and Equity Mutual Funds:
 - ____ I have a few stocks or small stock funds, and I am uncomfortable with the type of risk that stocks have.
 - ____ I am somewhat comfortable with this type of investment, it would be OK if more than 20% of my assets are in stocks or stock funds.
 - ____ I am very comfortable investing in stocks and stock mutual funds.

Risk Profile

1. Tolerance for Loss:

What percentage decline in the value of your portfolio could you observe over any one-year period without being really worried and uncomfortable?

- ____ Less than 5%
- ____ 5% to 10%
- ____ 10% to 15%
- ____ 15% to 25%
- ____ More than 25%
- 2. Ability to Handle Income Loss:

How long could you go without income before you had to use your long-term savings?

- ____ Less than 6 months
- ____ More than 6 months but less than a year
- ____ More than a year
- 3. Attitude Towards Risk:
 - ____ I must stick with my risk level
 - ____ I will take a bit more risk for more return
 - ____ I can take even more risk for a lot more expected reward